ing to have an experimental determination of the sign of this quantity. One such possible experiment has already been suggested by Chew.[6] We should remark that we have presented the nonrelativistic case only for the sake of comparison. However, even for the nonrelativistic case the eigenvalues are very small for $a_0>0$ unless unreasonably large cutoffs are introduced. Although there is a qualitative agreement between the various alternatives, quantitative results differ rather substantially. For this reason it would be interesting to consider the problem without putting the intermediate particles on the mass shell and compare the results with the above alternatives. On the other hand we have seen that the qualitative results of the Blankenbecler-Sugar alternatives applied to the Bethe-Salpeter equation are also obtained by introducing relativistic kinematics and phase-space factors in the Faddeev equation. A similar situation holds in the two particle case. There the Blankenbecler-Sugar rule applied to the two-particle Bethe-Salpeter equation gives the same result as the Lippmann-Schwinger equation with relativistic kinematics and phase-space factors.

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Nonet Meson Couplings to Baryons*

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A nonet coupling ansatz which requires zero couplings for the nonstrange baryons to the $\phi(1^{--}$, 1020 MeV) and $s(2^{++}$, 1525 MeV) $I=0$ mesons is investigated in the framework of a Regge-pole model for forward elastic scattering amplitudes at high energy. The analysis indicates that total cross sections are roughly consistent with the ansatz.

INTRODUCTION

An extension of the vector-meson nonet scheme relevant to the couplings of the vector mesons to baryons has recently been proposed[1] to account for (i) the relative suppression of backward ϕ/ω production of K^-p collisions, (ii) the proportionality of electric and magnetic form factors, and (iii) the isospin independence of the hard core in nucleon-nucleon scattering. The explanation of the above phenomena follows from a postulated $SU(3)$-invariant interaction Lagrangian in which all couplings of the nonstrange baryons to the physical ϕ meson are zero. Such an ansatz is made in the framework of a $SU(3)$ quark model in which the quark indices of the vector-meson nonet wave function are only allowed to couple *directly* to the quark indices of the baryon wave function. This anatz is independent of the f/d ratios of the $\bar{B}BV$ vertex and is therefore less restrictive than Lagrangians derived from higher symmetry schemes.[2]

In this article we investigate the validity of such a coupling ansatz for both the vector-meson and the tensor-meson nonets in the framework of a Regge-pole model[3] for forward elastic scattering amplitudes at high energy. Our analysis indicates that the total cross sections are roughly consistent with the ansatz.

VECTOR-MESON NONET

On the basis of mass formula and decay rates, the vector mesons [$\rho(760)$, $K^*(890)$, $\phi(1020)$, $\omega(783)$] are assigned to a $SU(3)$ nonet[4] $(V)_\alpha{}^\beta=(V_8)_\alpha{}^\beta+(1/\sqrt{3})V_1\delta_\alpha{}^\beta$ with the $\omega-\phi$ mixing specified by $\phi=(\sqrt{2}\phi_8-\omega_1)/\sqrt{3}$ and $\omega=(\phi_8+\sqrt{2}\omega_1)/\sqrt{3}$. The 3×3 matrix form of V may be written as

$$V=\begin{pmatrix}(\rho^0+\omega)/\sqrt{2} & \rho^+ & K^{*+}\\ \rho^- & (-\rho^0+\omega)/\sqrt{2} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & -\phi\end{pmatrix}. \quad (1)$$

Then the most general $SU(3)$-invariant interaction Lagrangians for the vector-meson Regge-pole residues at $t=0$ are

$$\begin{aligned}L_{VMM}&=\sqrt{2}\gamma_{MV}\langle M[V,M]\rangle,\\ L_{V\bar{B}B}&=\sqrt{2}\gamma_{NV}(f\langle\bar{B}[V,B]\rangle+(1-f)\langle\bar{B}\{V,B\}\rangle\\ &\qquad+\beta\langle V\rangle\langle\bar{B}B\rangle),\end{aligned} \quad (2)$$

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[1] H. Sugawara and F. von Hippel, Phys. Rev. **141**, 1331 (1966).
[2] In this paper we shall specifically deal with the $\bar{B}BV$ vertex which enters in the Reggeized vector-meson-exchange contribution to the *forward* elastic-scattering amplitude. In general both the conventional γ_μ and $\sigma_{\mu\nu}$ vector-meson nucleon couplings can contribute to the forward s-channel helicity nonflip Regge amplitude. Consequently, the f/d ratio for the $\bar{B}BV$ vertex at $t=0$ will represent a combination of both the electric- and magnetic-coupling contributions.
[3] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965); **16**, 545 (1966); Phys. Rev. (to be published).
[4] S. Okubo, Phys. Letters **5**, 165 (1963).

where M and B are the 3×3 matrices for the meson and baryon octets, respectively, and $\langle\ \rangle$ denotes trace over $SU(3)$ indices. The preceding ansatz implies that $\beta=2f-1$. We have previously argued that the vector-meson Regge-pole amplitudes should explain total-cross-section differences $\Delta_{AB}\equiv\sigma_t(\bar{A}B)-\sigma_t(AB)$ at high energy.[3] Consequently, in such a model we have

$$\tfrac{1}{2}\Delta_{\pi^+p}=2\gamma_{MV}\gamma_{NV}R_{\pi\rho p}(s)\,, \qquad (3)$$

$$\tfrac{1}{4}[\Delta_{K^+p}-\Delta_{K^+n}]=\gamma_{MV}\gamma_{NV}R_{K\rho p}(s)\,, \qquad (4)$$

$$\tfrac{1}{4}[\Delta_{K^+p}+\Delta_{K^+n}]=(4f-1)\gamma_{MV}\gamma_{NV}R_{K\omega p}(s)\,, \qquad (5)$$

$$\tfrac{1}{4}[\Delta_{pp}-\Delta_{pn}]=\gamma_{NV}{}^2R_{p\rho p}(s)\,, \qquad (6)$$

$$\tfrac{1}{4}[\Delta_{pp}+\Delta_{pn}]=(4f-1)^2\gamma_{NV}{}^2R_{p\omega p}(s)\,, \qquad (7)$$

where $R_{AVB}(s)$ contains the energy dependence of the Regge-pole amplitude[3]:

$$R_{AVB}(s)=\frac{\pi^{1/2}}{s^{1/2}q_{AB}(s)}\frac{\Gamma(\alpha_V+\frac{3}{2})}{\Gamma(\alpha_V+1)}\left(\frac{s-M_A{}^2-M_B{}^2}{s_V}\right)^{\alpha_V}. \qquad (8)$$

γ_{MV} is the residue factor of the VMM vertex. α_V is the $t=0$ intercept of the vector-meson Regge trajectory. The arbitrary scaling factor s_V is fixed to be $(1\ \text{BeV})^2$.

These equations for the Δ_{AB} involve five parameters: γ_{MV}, γ_{NV}, f, α_ρ, and α_ω. A least-squares fit of Eqs. (3)–(7) to 58 experimental measurements[5] yields $\chi^2=34.5$, indicating an adequate fit to the data. The values of the parameters were found to be

$$\begin{aligned}\alpha_\rho&=0.47\pm0.05\,,\\ \alpha_\omega&=0.37\pm0.05\,,\\ f&=1.94\pm0.04\,,\\ \gamma_N/\gamma_M&=0.5\pm0.1\,,\\ \gamma_N\gamma_M&=1.3\pm0.2\,.\end{aligned} \qquad (9)$$

The $\langle V\rangle\langle\bar{B}B\rangle$ coupling strength is $\beta=2f-1=2.9\pm0.8$. We conclude that the measured total-cross-section differences are consistent with a vector-meson Regge-pole model with zero $\phi\bar{N}N$ coupling and $f/d\approx-2$. Only an f part of the $V\bar{N}N$ coupling can be relevant to the electromagnetic nucleon charge form factor.

TENSOR-MESON NONET

The tensor mesons [$A_2(1310)$, $K^{**}(1430)$, $s(1525)$, $f(1250)$] also appear to exhibit nonet structure[6] on the basis of mass formula, with $(T)_\alpha{}^\beta=(T_8)_\alpha{}^\beta+(1/\sqrt{3})T_1\delta_\alpha{}^\beta$, and the s-f mixing[7] specified by $s=(\sqrt{2}s_8-f_1)/\sqrt{3}$ and $f=(s_8+\sqrt{2}f_1)/\sqrt{3}$.

The 3×3 matrix form of T may be written as

$$T=\begin{bmatrix}(A_2{}^0+f)/\sqrt{2} & A_2{}^+ & K^{**+}\\ A_2{}^- & (-A_2{}^0+f)/\sqrt{2} & K^{**0}\\ K^{**-} & \bar{K}^{**0} & -s\end{bmatrix}. \qquad (10)$$

Then the most general $SU(3)$-invariant interaction Lagrangian for the tensor-meson decays into two pseudoscalar mesons is

$$L_{TMM}=\sqrt{2}\gamma_{MT}(\langle M\{T,M\}\rangle+\epsilon\langle T\rangle\langle MM\rangle)\,. \qquad (11)$$

If the tensor mesons are also constructed from $q\bar{q}$ (quark-antiquark) states, then application of the preceding ansatz to the tensor nonet couplings requires $\epsilon=0$, since $\langle T\rangle$ does not couple *directly* to the quark indices of the pseudoscalar-meson octet wave function. This requirement of $\epsilon=0$ was previously found to be consistent with the tensor-meson decay rates.[6]

The total-cross-section sums $\Sigma_{AB}\equiv\sigma_t(\bar{A}B)+\sigma_t(AB)$ should be explained[8] in terms of the Regge-pole amplitudes corresponding to (i) a unitary singlet Pomeranchuk (P) of maximal strength, and (ii) the neutral zero-strangeness members of the tensor-meson nonet [A_2, K^{**}, s, f] whose trajectories are denoted by [R, Q, S, P']. The $SU(3)$ Lagrangian for the $t=0$ residues of the tensor-meson Regge poles with the baryon octet has a form similar to Eq. (2).

$$L_{TBB}=\sqrt{2}\gamma_{NT}(F\langle\bar{B}[T,B]\rangle+(1-F)\langle B\{T,B\}\rangle+\delta\langle T\rangle\langle\bar{B}B\rangle)\,. \qquad (12)$$

The nonet-coupling ansatz would imply $\delta=2F-1$. The phenomenological analysis of the Σ_{AB} in terms of the Pomeranchuk singlet and tensor nonet Regge poles proceeds in a similar fashion to the treatment of the Δ_{AB} except that deviations from exact symmetry are permitted in the Pomeranchuk couplings to $\pi\pi$ and $\bar{K}K$. Furthermore we do not *a priori* impose the ansatz that $\delta=2F-1$, inasmuch as the Σ_{AB} are relatively more sensitive to δ than the Δ_{AB} were to β; thus δ can be determined from the analysis of the Σ_{AB}. We use degenerate trajectory intercepts $\alpha_{P'}=\alpha_R=\alpha_S\equiv\alpha_T$ for this analysis. The theoretical equations for the Σ_{AB} involve eight parameters: the Pomeranchuk residues $(\Gamma_{\pi P},\Gamma_{KP},\Gamma_{NP})$, the tensor nonet residues $(\gamma_{MT},\gamma_{NT})$, α_T, F, and δ. We determine these eight parameters by minimizing χ^2 in a statistical fit to 35 measurements of

[5] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, M. L. Read, and R. Rubinstein, Brookhaven National Laboratory Report No. 9410, 1965 (unpublished); R. H. Phillips (private communication); A. Citron *et al.*, Phys. Rev. Letters **13**, 205 (1964); W. F. Baker *et al.*, in *Proceedings of the Sienna International Conference on Elementary Particles*, edited by G. Bernardini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963), Vol. I, p. 634; A. N. Diddens *et al.*, Phys. Rev. Letters **10**, 262 (1963); G. von Dardel *et al.*, *ibid.* **7**, 127 (1961); S. J. Lindenbaum *et al.*, *ibid.* **7**, 352 (1961); G. von Dardel *et al.*, *ibid.* **8**, 173 (1962); W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965); W. F. Baker *et al.*, *ibid.* **129**, 2285 (1963); S. J. Lindenbaum *et al.*, Phys. Rev. Letters **7**, 185 (1961); G. von Dardel, *ibid.* **5**, 333 (1960).

[6] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[7] In our analysis we use the theoretical mixing angle $\sin\theta=1/\sqrt{3}$ ($\theta\approx35.3°$) for both the vector and tensor nonets. The mixing angles calculated from observed masses are $\theta=40°$ for the vector nonet and $\theta=30°$ for the tensor nonet (Ref. 6).

[8] V. Barger and M. Olsson, Phys. Rev. (to be published).

the Σ_{AB} given by Galbraith *et al.*[5] Our solution yields $\chi^2=22$ indicating an adequate fit to the data. The fitted parameters α_T, F, and δ were determined to be

$$\alpha_T=0.39\pm0.24\,,\quad F=2.0\pm0.6\,,\quad \delta=2.3\pm1.1\,. \tag{13}$$

Consequently the ansatz condition $\delta=2F-1$ is to be compared with the fitted values $\delta=2.3\pm1.1$ and $2F-1=3.0\pm1.2$. Hence the total-cross-section data indicate approximate validity of the ansatz for the tensor nonet couplings. If we constrain $\delta=2F-1$ for the fit (which decouples S from the nucleons), then we obtain

$$\alpha_T=0.57\pm0.22\,,\quad F=2.2\pm1.0\,, \tag{14}$$

and thus

$$\delta=2F-1=3.4\pm2.0\,.$$

However, since this value of α_T is somewhat larger than the value $\alpha_R\approx0.4$ determined[9] from the $\pi^-p\to\eta n$ differential-cross-section data, it appears that S is approximately but perhaps not entirely decoupled from $\bar{N}N$.[10] More precise measurements on the Σ_{AB} will permit considerable refinement of this analysis (such as removal of the constraint $\alpha_R=\alpha_{P'}=\alpha_S$).

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[9] R. J. N. Phillips and W. Rarita, Phys. Letters **19**, 598 (1965).

[10] With $\delta=2F-1$ we have not been able to find a χ^2 minimum for $\alpha_R<\alpha_S-0.1$. However, this does not exclude the possible existence of such solutions with an acceptable χ^2. The decoupling of the physical particle s depends somewhat on the precise value of the mixing angle (Ref. 7). An independent test of the $s\bar{N}N$ decoupling hypothesis would be experimental observation of suppression of $K^-p\to\Lambda s$ relative to $K^-p\to\Lambda f$ at backward angles.

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Separable Two-Body Potentials for Multiparticle Scattering*

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Separable potential forms which lead to on-energy-shell partial-wave transition amplitudes for two-body scattering which have analyticity properties very similar to those proved in local potential theory are studied. An explicitly energy-dependent potential form is shown numerically to give a better approximation to the amplitude from a local Yukawa potential than the best previous form, for $l>0$ in the scattering region. Some remarks are made about the applicability of these potential forms to multiparticle scattering calculations.

I. INTRODUCTION

RECENT papers on multiparticle scattering[1] have stressed the need for a good separable approximation to the two-particle transition amplitude. In particular, the introduction of a separable approximation for the off-energy-shell two-body amplitude in the integral equations for the three-body transition amplitude reduces the dimensionality of these integral equations and makes it possible to solve them on a computer.

Not long ago, Noyes[2] and Kowakski[3] gave a separable approximation to the off-shell two-body amplitude derived from local potential theory. However, Basdevant[4] points out that their approximation has cuts for $k^2<0$ (we take $\hbar=2m=1$, so that k^2 is the energy variable) that are not present in the full off-energy-shell two-particle transition amplitude defined by the Lippmann-Schwinger equation. He further points out that these cuts will be in regions of k^2 over which one must integrate in the three-body problem and that they can lead to complex eigenvalues in the three-body bound-state region. Finally, numerical solutions of the equations of Noyes and Kowalski for a Yukawa potential have shown that the term neglected in their approximation to $t_l(p,p',k)$ is comparable in size to the term retained when p and p' differ from k by an amount comparable to k.

In the light of these developments, the spirit of the present note is as follows:

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[1] C. Lovelace, Phys. Rev. **135**, B1225 (1964); B. S. Bhakar and A. N. Mitra, Phys. Rev. Letters **14**, 143 (1965); R. Aaron, R. D. Amado, and Y. Y. Yam, *ibid.* **13**, 574 (1964); M. Bander, Phys. Rev. **138**, B322 (1965); J. L. Basdevant, *ibid.* **138**, B892 (1965); J. H. Hetherington and L. H. Schick, *ibid.* **139**, B1164 (1965).

[2] H. Pierre Noyes, Phys. Rev. Letters **15**, 538 (1965).

[3] K. L. Kowalski, Phys. Rev. Letters **15**, 798 (1965).

[4] J. L. Basdevant (Centre de Recherches Nucléaires, Strasbourg), private communication to H. Pierre Noyes.